A Fast Solution of Rotor Harmonic Losses in Cage Induction Motors by Time-Stepping Finite Element Method

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Time-stepping finite element method has been widely used to calculate the rotor flux density and the bar current density for the rotor iron and copper losses in cage induction motors. The conventional calculation of these losses usually needs a full slip-cycle simulation of the rotor flux density and the bar current density waves to include the effect of fundamental and harmonic components. Under load conditions this simulation can be extremely expensive in CPU time. This paper proposes a new technique of simulation and post-processing that significantly reduces the calculation overhead. The proposed technique retrieves the full slip-cycle waves by utilizing both the time-varying and the spatial information of the flux density and current density, and saves the CPU time by a ratio depending on the number of rotor slots per pole pair. The more rotor slots per pole pair the induction motor has, the less CPU time the technique needs. The rotor flux density and current density harmonics of a 5.5 kW induction motor are extracted by the proposed technique, and compared with those extracted by the conventional method for validation.

***Index Terms*—Induction motors, rotor harmonics, rotor losses, time-stepping finite element method**

I. Introduction

The rotor loss of a cage induction motor consists of iron loss and copper loss that are caused by both fundamental and harmonic flux density and bar current. Time-stepping finite element method (T-S FEM) is a widely used to predict the rotor flux density and bar current density waves which are post-processed for the iron and copper losses [1], [2]. The conventional prediction of rotor losses under load conditions involves a large expense in CPU time in the process of the time-stepping finite element simulation as it must simulate a full slip cycle of flux density and bar current density waves in order to take into account all the information of the harmonic, as well as the slip-frequency (fundamental) components. This expense can be extremely large when simulating the load operation of an inverter-fed cage induction motor as very small time steps have to be chosen to cover the high switching frequency of the pulse width modulation (PWM) supply. This drawback may limit the usage of T-S FEM in day-to-day design of cage induction motors.

As the rotor flux density in an induction motor is a wave travelling through the fully symmetric rotor structure, the flux densities at a certain position in different rotor teeth present as time-varying waveforms with equal fundamental and harmonic magnitudes, respectively, and with different phase angles. The induced bar current densities at a certain position in different rotor bars present in the same manner. The conventional method has to simulate a full slip cycle of the flux density and the bar current density waves as it utilizes only their time information. The full slip-cycle simulation may not be necessary if the spatial information can also be utilized. For example, by a linear regression with only one supply cycle of the simulated flux densities at multiple teeth, the magnitudes of the fundamental and harmonic flux densities are all found for calculation of iron losses [1]. This paper proposes an alternative technique that determines the rotor fundamental and harmonic flux densities and bar current densities with the simulation of a period inversely proportional to the number of rotor slots per pole pair. This technique is applied to the simulation and post-processing for the rotor loss of a 5.5 kW cage induction motor, and the calculated harmonics are compared with those given by the conventional method for validation of the proposed technique.

II. Description of the Proposed Technique

The flux density of the rotor in an induction motor can be found by considering the magneto-motive force (mmf) and permeance harmonics. In the rotor reference frame, the flux density harmonics seen by the rotor can be expressed by

$$B_{\mathrm{m}}\cos\left\{(np+qN_{\mathrm{s}})\theta_{\mathrm{R}}+\phi-\left[1-\frac{1-s}{p}\left(np+qN_{\mathrm{s}}\right)\right]\omega_{\mathrm{e}}t\right\}$$
$$+B_{\mathrm{m}}\cos\left\{(np-qN_{\mathrm{s}})\theta_{\mathrm{R}}+\phi-\left[1-\frac{1-s}{p}\left(np-qN_{\mathrm{s}}\right)\right]\omega_{\mathrm{e}}t\right\},$$
$$n = 1, 5, 7, 11, 13 \ldots, q = 0, 1, 2 \ldots \text{ [1]} \quad (1)$$

where p is the number of pole pairs, N_{s} is the number of stator slots, ϕ is a phase angle, ω_{e} is the electrical frequency of the stator current, s is slip, θ_{R} is angle referred to the rotor, n and q are the harmonic orders of mmf and permeance, respectively, B_{m} is magnitude of flux density harmonic, which is a function of n, q, and the position, i.e., θ_{R} and the radius on the rotor. The expression of induced bar current density has the same form with (1).

As shown in (1), the flux density and the bar current density waves at a certain position in different rotor teeth and bars have equal magnitude and different phase angle, respectively. In other words, a period of simulated flux densities and current densities at a certain position in different rotor teeth and bars reflect different phase of the travelling waves, respectively. This spatial information can be utilized for a shorter period of simulated time. As shown in Fig. 1, a single element at identical position is chosen in each rotor teeth and bar, respectively, over one pole pair pitch. As shown in Fig. 2, the flux density and current density waves in these elements should present a phase shift of $360p/N_r$ degree, where N_r is the number of rotor slots. The whole travelling waves of the flux density and the current density can be retrieved by incorporating the N_r/p calculated wave segments of $p/(sfN_r)$ seconds from the chosen elements. By applying FFT to the waves from these elements at different positions in the rotor teeth and bars the fundamental and the harmonics can be extracted for calculations of iron and copper losses, respectively. The advantage of the proposed technique is that the simulated time is only p/N_r times that of the conventional method, and the incorporated waves automatically include the high-order harmonics, which could be partially lost by the method proposed in [1] due to the low spatial sampling frequency, and have to be estimated by linear regression.

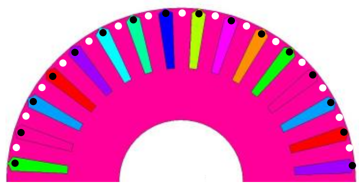

Fig. 1. Sample elements in rotor teeth and bars over one pole pair pitch

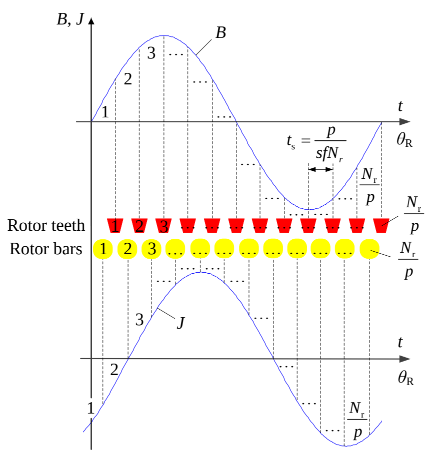

Fig. 2. Retrieving the waves of rotor tooth flux density and bar current density

III. An Example of Calculation and Measurement

The proposed technique is used to calculate the rotor loss of a 5.5-kW, 220-V, 50-Hz, 4-pole cage induction motor, which has 36 stator slots and 32 rotor slots. 16 sample elements are chosen on the top of rotor teeth and bars, respectively, over one pole pair pitch as shown in Fig. 1. The load operation with a slip of 0.04 is simulated by T-S FEM for 1/32 seconds, as described in section II. Fig. 3 shows the calculated radial flux density of a sample element on the top of a rotor tooth, by the conventional full slip-cycle simulation and by the proposed technique, respectively. Fig. 4 shows the calculated current density of a sample element on the top of a rotor bar, by the conventional full slip-cycle simulation and by the proposed technique, respectively. The flux density harmonics given by the proposed technique and by the conventional method are detailed in Table I for comparison, which shows close or acceptable agreement. The detailed information of current density harmonics are omitted due to the limited space.

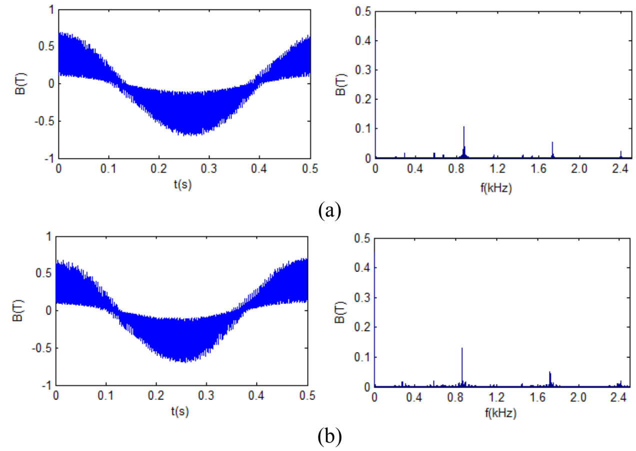

Fig. 3. The calculated radial flux density on the top of a rotor tooth. (a) the waveform and harmonics by the conventional method. (b) the waveform and harmonics by the proposed technique.

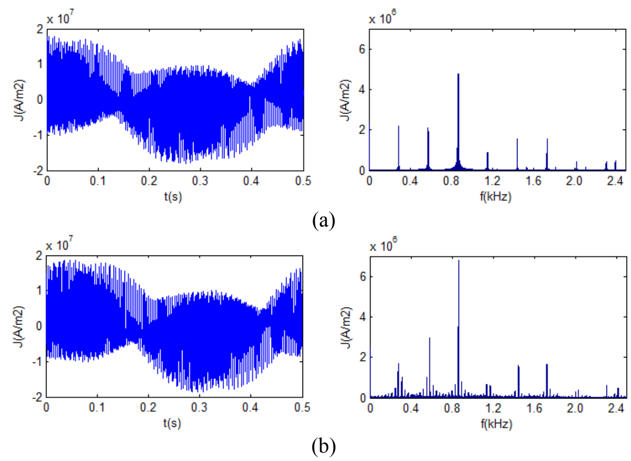

Fig. 4. The calculated current density on the top of a rotor bar. (a) the waveform and harmonics by the conventional method. (b) the waveform and harmonics by the proposed technique.

TABLE I
Calculated Flux Density Harmonics in Rotor Teeth

Harmonic order and frequency	Conventional Method (T)	Proposed Method (T)
Fundamental, 2Hz	0.45	0.44
286Hz (*n*=7, *q*=0)	0.0175	0.018
578Hz (*n*=7, *q*=1)	0.0215	0.0163
Approximately 862Hz to 866Hz (*n*=1, *q*=1, fundamental slot harmonic)	0.132	0.107
Approximately 1726Hz to 1730Hz (*n*=1, *q*=2, the 2nd slot harmonic)	0.0498	0.0543

IV. Conclusion and Future Work

For the rotor flux density and current density harmonics in cage induction motors the proposed simulation and post-processing method can significantly reduce the calculation overhead with acceptable accuracy. The proposed method will be used as a fast solution of rotor losses in the future work.

References

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